

**THE DEVELOPMENT OF TWO-DIMENSIONAL  
OBJECT IDENTIFICATION TECHNIQUES**

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## Abstract

This report marks the end of a one year of an anticipated three year effort involving undergraduate and graduate students in the study of methods for numerically identifying objects according to shape in two dimensions. The method is based upon comparing the unit gradient of an observed object and the unit gradient of a standard object over a specified range of points. The manner in which the gradients are compared forms the basis of a shape recognition scheme, which is then applied to simple closed plane figures. The gradient based method is calibrated by using various distorted objects in comparison with a set of standard reference objects. The use of pattern recognition techniques for computer identification of two-dimensional figures was to be investigated during the second and third years of this project.

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## Chapter 1

### INTRODUCTION

The following items were proposed and accomplished during the first year of this grant:

1. Problem Study Phase;
2. Initial Shape Metric Formulation;
3. Two Dimensional Shape Identification Group Events, and
4. Formulation of a Two-Dimensional Object Identification System.

In the Problem Study Phase, the definition of shape was refined. The refined definition lead to a mathematical definition of shape. From the mathematical definition of shape an initial shape metric was developed. The concepts behind the initial shape metric was presented to several groups in academia and in industry. The scientific interaction with colleagues lead to the formulation of a rudimentary object identification system.

The second year of this grant proposed a continuation that would have involved, the establishment of a Two-Dimensional Object Identification System (TOIS) in software. In this phase, two dimensional figures were to be identified using algorithms that tolerated internal structures. Following the success of the software implementation of the TOIS, the third year goal was to produce a microprocessor based equivalent.

## Chapter 2

### PROBLEM STUDY PHASE

The problem study phase of this research lasted approximately from April 1, 1988 until August 14, 1988. During this time, the research group narrowed down some basic definitions concerning how objects were identifiable. In these discussions, it was concluded that objects could only be identified if they could be distinguished from the environmental background. The very act of discernment lead to the premise that in identifying any object, a comparison is done on the object with respect to the environment. It was concluded that the base action in all scientific activities involved a comparison. Inherent in all situations where comparisons are made, there must be (1) an observer, (2) an observable, (3) a reference, and (4) an agent (Figure 2.1) [1, pp.47-48]. The observable is compared with a reference object. Both reference and observable have a characteristic in common and of interest. The agent is a carrier of information concerning the characteristic present in the observable, relative to some reference object. The information carried by the agent is relayed to the observer. The observer is an entity capable of assimilating the relayed information.

In comparing two items, one may look either for similarities or differences depending upon pre-determined characteristics of interest. In the project performed by the University, the characteristic of interest is shape. In determining a characteristic of interest, one must first start with its basic definition.

The definition of shape is nebulous at best. One popular dictionary defines shape as a mode of existence or a form of being having identifying features [2]. While this may be a fine definition for human consumption, it does not avail in terms of forming a numerical algorithm for comparing shapes. This lack of a concrete definition of shape, lead the research group to seek a mathematical of shape. The formulation of a mathematical statement of shape marked the end of the problem study phase and the beginning of the shape metric phase.

#### 2.1 Initial Shape Metric Formulation

In seeking a mathematical solution to shape determination, the research group investigated the properties of a general surface. Given a general surface, the unit normal

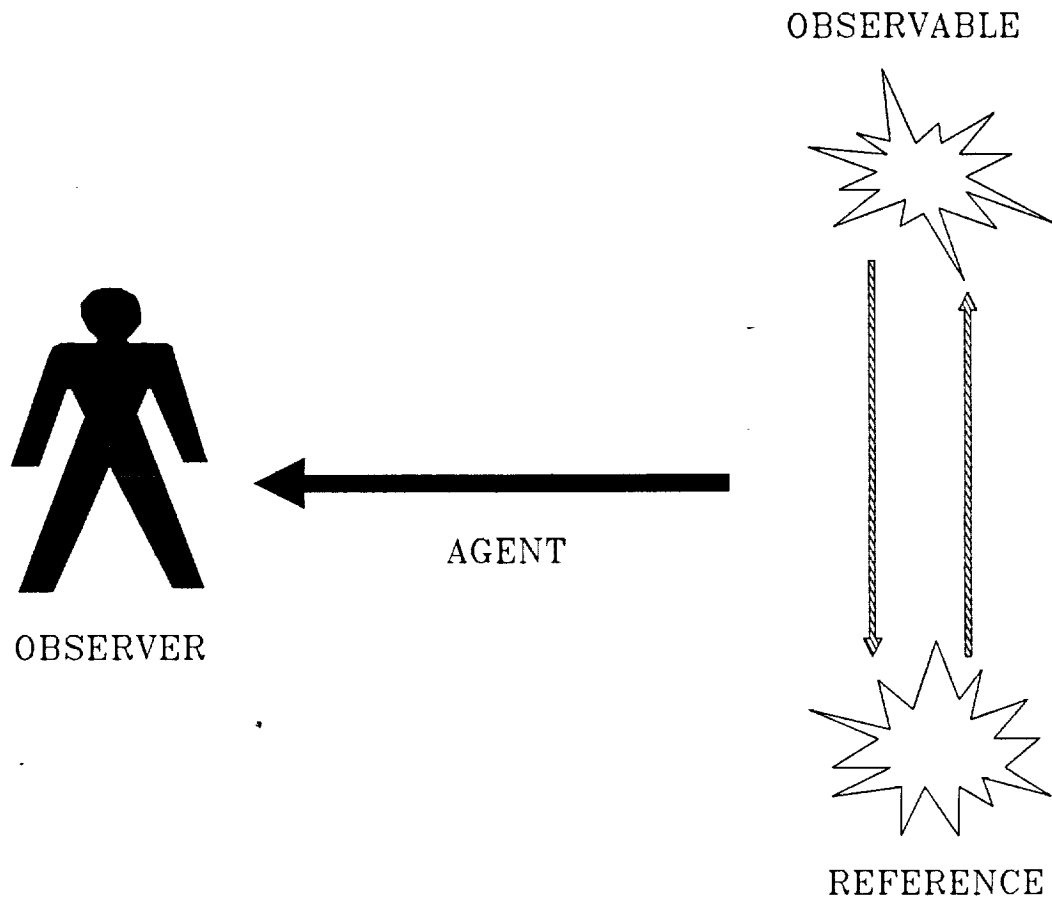


Figure 2.1: Idealized Measuring Relationship.

at every point to this surface is assumed to be a feature of shape. If the gradient to a curve is known at all points along the curve, then a curve of similar shape can be constructed from the set of gradients [3, pp. 650-658]. The constructed curve may be a translate, rotate, enlargement or reduction of the original curve, but it will have the same shape content. During discussions, the group decided to use the unit normal to the surface at a set of points in comparing shapes, since it appears to capture the shape of the surface of interest. In addition to capturing shape, there is a wealth of literature concerning the use of gradients in determining the direction of the surface. The next step was to develop a numerical method for comparing functions in two-space (Refer to Appendix A.) This step was successful and it resulted in a uni-dimensional metric that allowed the comparison of functions in two-space. The draw-back of this metric is that it does not allow for the presence of internal structure. Through successive application of this metric, at best the outline of a two dimensional object may be compared. Some applications of the uni-dimensional metric is shown in Appendix B.

The next step of research involved devising a method that would overcome, the mandatory successive application of the uni-dimensional metric. This search lead to the development of the polar shape metric (Refer to Appendix C.) The polar shape metric allowed the comparison of closed curves in two-space. The comparison was performed by:

- centering both objects at the origin,
- constructing gradients to the curves at the points intersected by rays from the origin at differing angle between 0 and  $2\pi$ ,
- on a point-by-point basis, carrying out a comparison analogous to the comparison used in the uni-directional curve shape metric.

Here the curves are assumed to be relatively smooth with a finite number of discontinuities (places where the gradient is undefined.)

The research group presented the initial ideas concerning object determination via shape to the scientific community at student and professional levels.

## 2.2 Shape Identification Group Events

During the first phase, the research group participated in the following presentations or events:

1. Lebby, G. L., "*Recognition of Plane Figures Using a Gradient-Based Shape Metric*," Engineering System and Technology Division Symposium, Minnesota Mining and Manufacturing, September, 1988.

2. Lebby, G. L., Matherson, J.M., and E. E. Sherrod, *"Two Dimensional Object Detection Technique Using a Gradient Based Metric,"* Abstract, 1989 IEEE Symposium on System Theory.
3. Matherson, J.M., Sherrod, E.E., and G.L. Lebby, *"Two-Dimensional Object Identification Using a Gradient Based Metric Technique,"* Presentation, NASA-Langley 1988 HBCU Workshop.

Refer to Appendix D for a sample of overhead transparencies used in the presentations.



## Chapter 3

### Formulation of the TOIS

In defining the Two-Dimensional Object Identification System (TOIS) the following system types are used:

- Image\_Palette - an area used for storing the descriptions of multiple objects in a scene.
- Object\_Descriptor\_Vector - a set of descriptions pertaining to objects identified in an Image\_Palette.

Refer to Figure 3.1. The TOIS receives input via an Image\_Palette that describes the two dimensional scene of interest. If the Image\_Palette is empty, then the system resets and awaits another set of input. If the Image\_Palette is not empty, then the system recursively performs the following modular activities:

1. Locate\_Object
2. Identify\_Object
3. Generate\_Descriptor

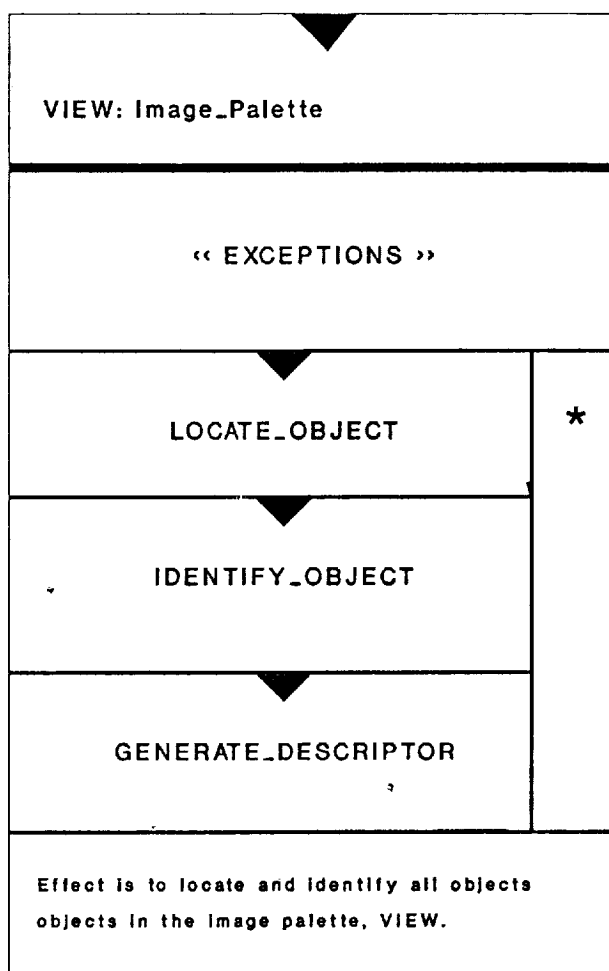
The effect of the TOIS is to locate and identify all objects in a specified image palette.

#### 3.1 Description of the Locate\_Object Module

The purpose of the Locate\_Object module is to locate the next object in the specified Image\_Palette (Figure 3.2). Initially, the Locate\_Object module is set to Object\_Zero. Object\_Zero is a special object that lets Locate\_Object know that there are no objects located in the current Image\_Palette. Locate\_Object will perform the following for a non-empty Image\_Palette:

1. Extract\_Next\_Object
2. Remove\_Object

TOIS:

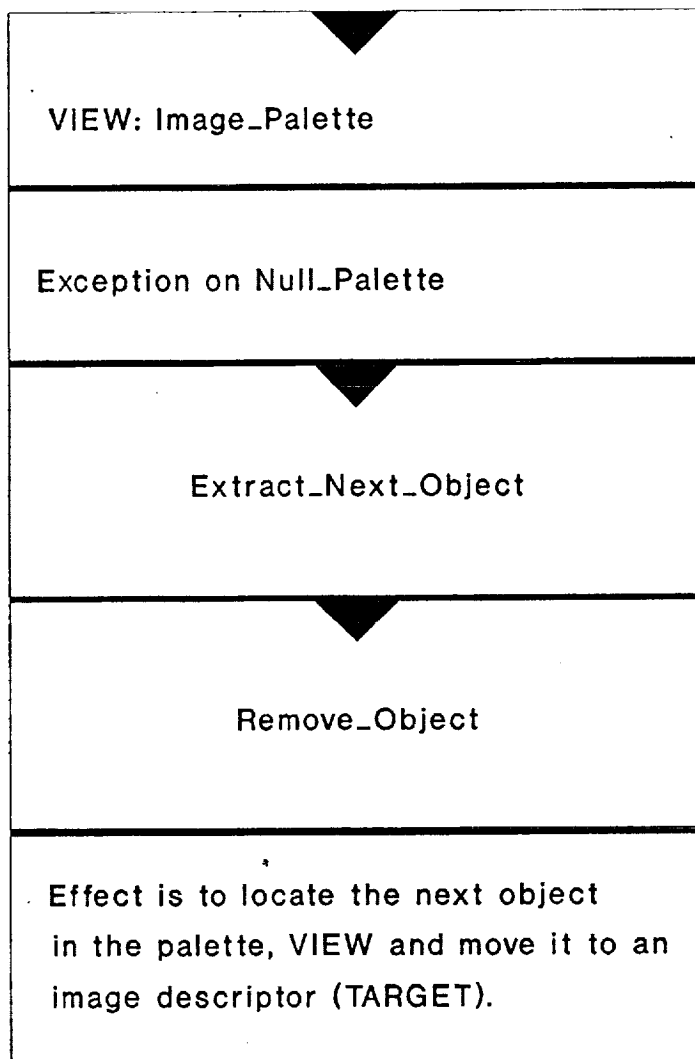


:RETURN

object\_descriptor\_vector

Figure 3.1: Two-Dimensional Object Identification System.

LOCATE\_OBJECT:



:RETURN

Image\_Descriptor

Figure 3.2: Locate.Object Module.

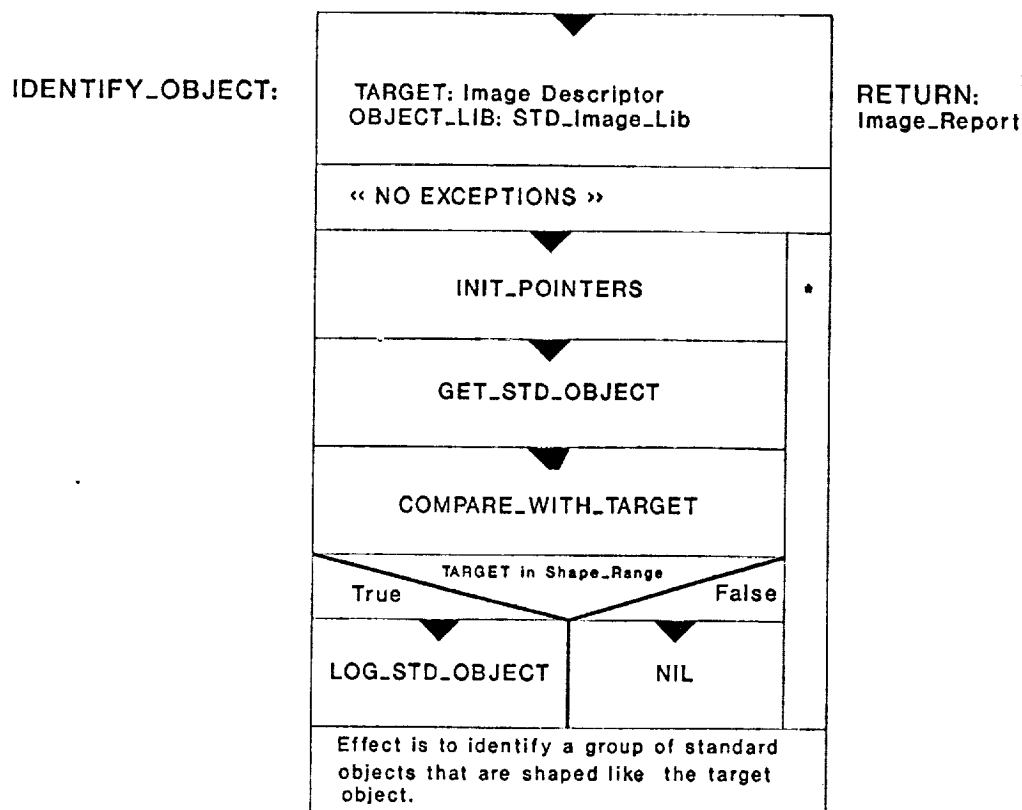


Figure 3.3: Identify\_Object Module.

The Extract\_Next\_Object module moves the system's attention to the next object in the Image\_Palette. The critical image description and location are recorded in the current Image\_Descriptor.

The Remove\_Object module operates on the current Image\_Palette to mask or delete the current object. The current object is marked as viewed, so that it will not be reselected for identification. The overall effect of the Remove\_Object module is to delete the current object.

### 3.2 Description of the Identify\_Object Module

The purpose of the Identify\_Object module is to compare the current object description pointed to in the Image\_Descriptor to objects described in a library of standardized images (Figure 3.3) The following additional types are used in the definition of the Identify\_Object module:

- STD\_Image\_Lib - this is a library of standardized images used to identify the unknown target image.
- Image\_Report - this is a report of the comparisons made between the unknown target and each of the known reference objects from the standard library.

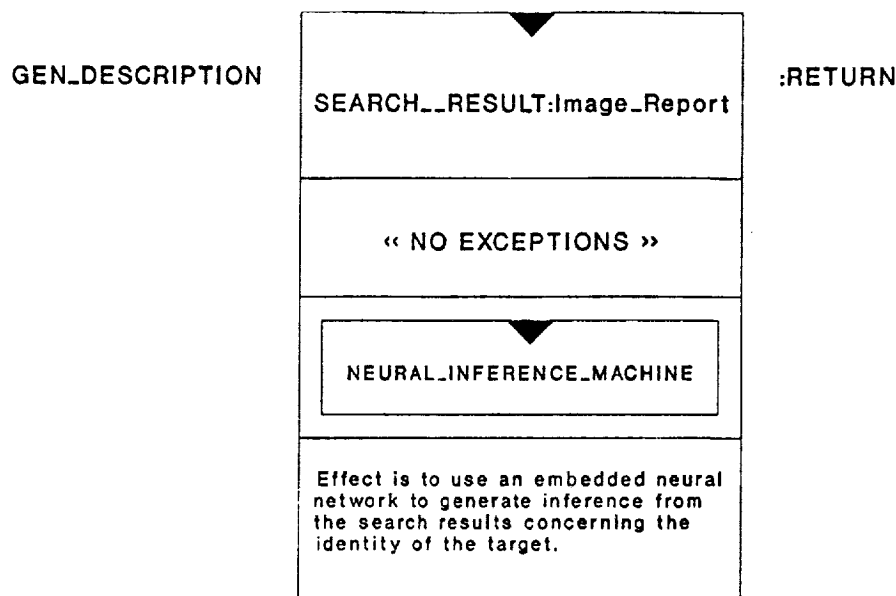


Figure 3.4: Gen\_Description Module.

The Identify\_Object module takes as input, the Image\_Descriptor and the STD\_Image\_Lib. The module INIT\_POINTERS, initializes appropriate data pointers so that the current unknown target is referenced throughout the identification and is compared with the k-th image in the standard library where k varies from the first to the last standard image. The Get\_STD\_Object module fetches the k-th standard object from the library. The Compare\_With\_Target module does a shapewise comparison of the unknown target with the k-th standard object. If the comparison of the two objects are within an acceptable range (Shape\_Range) then control is passed to a module called Log\_STD\_Object. The module Log\_STD\_Object is responsible for creating entries in the Image\_Report concerning possible library matches.

The overall effect of the Identify\_Object module is to identify a subset of objects from the library of standardized objects that are similar in shape to the unknown target object. The information concerning the comparison is recorded in the Image\_Report.

### 3.3 Description of the Gen\_Description Module

The purpose of the Gen\_Description module is to generate a description of the unknown target object in terms of pre-defined standard objects (Figure 3.4.) The core

of this module is a neural inference machine that iterprets the Image\_Reports and makes inferences concerning the identity of the target object based upon shape. The information returned by this module is an indexed description of the target object in terms of probable matches with a subset of standard objects. The neural inference machine development and training is part of the goals for the second year of this project. The effect of the Gen\_Description module is to generate an Object\_Description\_Vector entry concerning the target object's identity. The identity is inferred from the Image\_Report.

## Chapter 4

### Summary

For the first year, a gradient based method for comparing curves in two-space has been developed and refined. The research group has presented the concepts behind the gradient shape metric to the scientific community. A system for two-dimensional object identification (TOIS) has also been proposed and discussed. The completion of these items mark the end of Phase I.

Phase II entails establishing the TOIS in software. In this phase, the identifying algorithms will be able to handle two dimensional object of increased complexity. The goal of Phase II will be the complete identification of objects in a scene.

Phase III involves the production of a microprocessor based equivalent of the TOIS. The goal of this phase will be to recognize objects in real time. Human recognition of a scene will be employed as a measure of the hardware based TOIS.

## Appendix A

### Shape Metric Development

#### A.1 FUNCTION COMPARISON IN TWO-SPACE

Assume that there exist two functions in two-space, called  $y(t)$  and  $x(t)$  (Figure A.1.) Further assume that there exist corresponding sets of gradients to the functions  $x(t)$  and  $y(t)$  called  $C_x$  and  $C_y$  respectively. There is a real function that is defined for all curves  $C_x$  and  $C_y$  that are elements of the set  $\Psi$  and it satisfies the following properties [4] :

1.  $\mathcal{L}(C_x, C_y) = \mathcal{L}(C_y, C_x)$  (symmetry)
2.  $\mathcal{L}(C_x, C_y) \geq 0$  (nonnegativity)
3.  $\mathcal{L}(C_x, C_y) = 0$  iff  $C_x$  and  $C_y$  are equal (nondegeneracy condition).
4.  $\mathcal{L}(C_x, C_y) \leq \mathcal{L}(C_x, C_z) + \mathcal{L}(C_z, C_y)$   
for all elements  $C_x, C_y$ , and  $C_z$  that belong to the set  $\Psi$  (triangle inequality).

If the above is true, then elements of  $\Psi$  are called points in the metric space  $(\Psi, \mathcal{L})$  with  $\mathcal{L}$  being a metric or distance function defined over  $\Psi$ .

At this point, measuring has been covered in general, also the definition of a metric has been established. The topic of curve measuring will be motivated in the following sub-section. In this section, the observable and the standard reference have been restricted to being continuous functions in two-space. This accommodates studying daily PSL curves, since the horizontal axis represents the hours of the day and the vertical axis represents the magnitude of the PSL in units of power (in this case the units of power are megawatts).

Further assume that we are interested in comparing the two curves  $x(t)$  and  $y(t)$  over a closed interval  $[a, b]$  where  $y(t)$  and  $x(t)$  are continuous and have a finite gradient defined over the closed interval  $[a, b]$ . In principle, the two curves must be compared at each point in time  $t$ . Intuitively, the comparison must ignore translations in the vertical direction, however, translations in the horizontal directions will not be accommodated for. This can also be easily thought of as comparing like points in



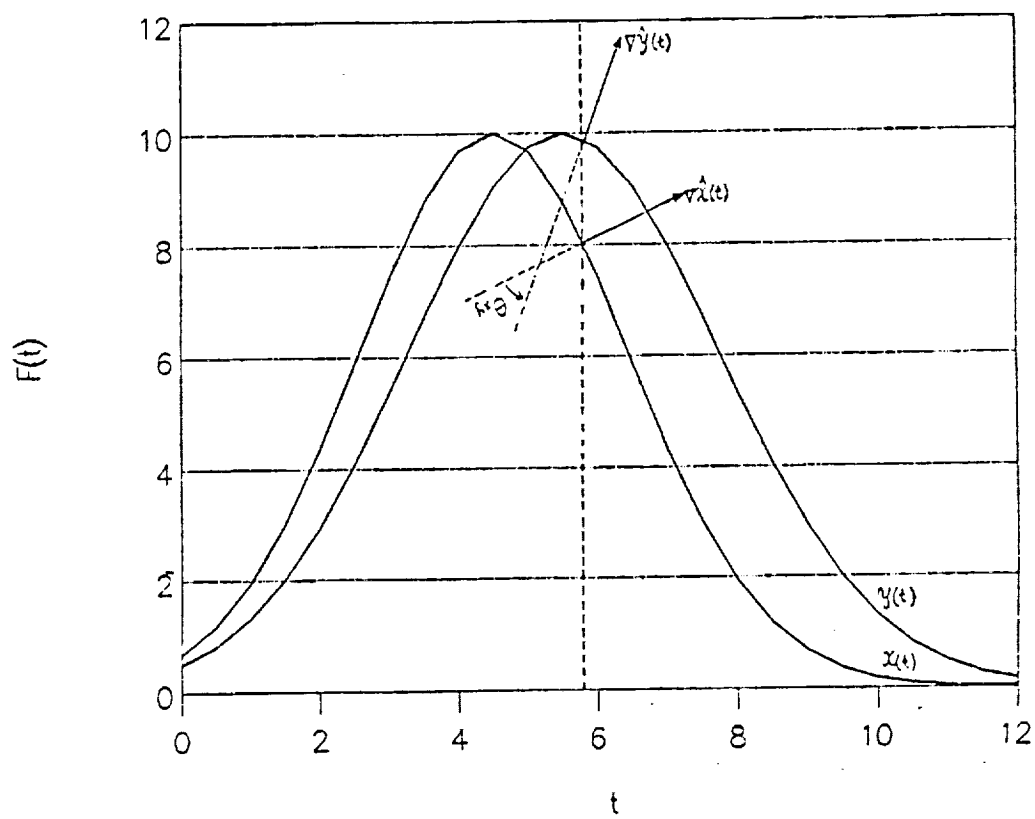


Figure A.1: Gradient Comparison of Two Curves at a Point.

time, since horizontal shifts will cause shifts in time. Vertical shifts cause shifts in magnitude and this does not cause the problems that a horizontal shift would. The central assumption concerns the equality of shape over a closed interval. Two curves  $x(t)$  and  $y(t)$  will be considered equally shaped over the closed interval  $[a,b]$  if and only if (iff) the gradients constructed at the point  $t_0$  to  $x(t_0)$  and  $y(t_0)$  are equal for all points  $t_0 \in [a,b]$ . From this assumption, it can be gathered that the curves are either equal or they are not equal over a specified closed interval. Yet, there exist a desire to express not only the equality of the two curves  $x(t)$  and  $y(t)$ , but to express a measure of similarity or dissimilarity in the case that the two curves are not equal. Gradient similarity is represented by use of the absolute value of the scalar inner product between the gradient vectors at a point  $t_0$ . Similarly, the gradient dissimilarity is represented by using the magnitude of the vector product between the two gradient vectors at a point  $t_0$ . Referring to Figure A.1, assume that each constructed gradient is normalized. Normalization assures that the absolute value of the scalar inner product or the magnitude of the vector product between any two vectors lie between zero and one. Normalizing the gradient vectors on each of the curves at each point in time serves to weight each comparison between  $x(t)$  and  $y(t)$ . This comparison can be one similarity or dissimilarity. When dealing with similarity, assume that a gradient  $\hat{\nabla}x(t)$  is compared to another gradient  $\hat{\nabla}y(t)$  by performing the scalar inner product  $\hat{\nabla}x(t) \cdot \hat{\nabla}y(t)$ . Since the gradients involved are normalized, in the case that  $\hat{\nabla}x(t)$  equals  $\hat{\nabla}y(t)$ , the scalar inner product will result in a value of one. The direction in which the gradient points (the gradient is a vector quantity, having both direction and magnitude) is sometimes termed the direction of the surface. In this case the surfaces involved are the curves themselves. The absolute value of scalar inner product between two vectors in general takes on values between zero and one inclusive. If the vectors point in the same direction, the inner product will be one. If they are perpendicular, the inner product will be zero. Form a number  $\lambda^*(t_0)$ , such that

$$\lambda^*(t_0) = | \hat{\nabla}x(t_0) \cdot \hat{\nabla}y(t_0) |$$

for each point  $t_0 \in [a,b]$ . This lays the basis for a transformation that is in a sense a reversed measure since it increases with the similarity of the functions  $x(t)$  and  $y(t)$ . If  $x(t)$  has the same shape as  $y(t)$  then  $\lambda^*(t)$  would always be at it's maximum value of 1. Summing  $\lambda^*(t)$  for all  $t$  would yield an average value of 1. If the sum of  $\lambda^*(t)$  was a distance function then it should be zero when  $x(t)$  equals  $y(t)$  and not one. Since it is desirable to obtain not only an index of closeness, but a bonafide measure, the alternate transformation called "dissimilarity" will be used. Intuitively, dissimilarity will tend to zero as the curves  $x(t)$  and  $y(t)$  tend to each other. In using dissimilarity as a candidate measure, the arguments and assumptions made regarding the gradient normals are retained. When dealing with dissimilarity, assume that a gradient  $\hat{\nabla}x(t)$  is compared to another gradient  $\hat{\nabla}y(t)$  by performing the vector product  $\hat{\nabla}x(t) \times \hat{\nabla}y(t)$ . The magnitude of the vector product between two unit normal vectors takes on values between zero and one inclusive. If the vectors point

in the same direction, the result will be zero. If they are perpendicular, the vector product will be one. Form a number  $\mathcal{L}(t_0)$ , such that

$$\lambda(t_0) = | \hat{\nabla}x(t) \times \hat{\nabla}y(t) |, \quad (\text{A.1})$$

for each point  $t_0 \in [a, b]$ . Next, sum all the numbers  $\lambda(t_0)$ , for each  $t_0$  in the closed interval  $[a, b]$  and then take the average of this sum. Call this resultant value  $\mathcal{L}(\dot{x}(t), \dot{y}(t))$  is

$$\mathcal{L}(\dot{x}(t), \dot{y}(t)) = \int \lambda(t) dt \quad (\text{A.2})$$

where the interval  $[a, b]$  is assumed to have a non-zero length and  $b$  is greater than  $a$ . This value  $\mathcal{L}(x, y)$  is hereafter referred to as the distance between the shape of  $x(t)$  and the shape of  $y(t)$  over the closed interval  $[a, b]$ . This operation  $\mathcal{L}$  is claimed to be a metric for the characteristic "closeness of shape" in shape space. The next section will fully define this newly mentioned metric space, "shape space" and proceed with proofs that  $\mathcal{L}$  is a metric that will measure distance in that space.

## A.2 DISTANCE IN SHAPE SPACE

It is claimed that shape like any other characteristic that is observable can be measured and given a unique number which determines a relationship between one object with respect to another. A major concern is with comparing modeled PSL curves with that of the actual PSL curve. These curves are in two-space, so the proof will deal with curves in two-space, although the proof could be extended to include  $n$ -space, where  $n \geq 2$  and  $n$  is an integer. Further assume that we are interested in obtaining the average difference in shape between two curves  $x(t)$  and  $y(t)$  over a non-zero length interval of time where  $a \leq t \leq b$ . In addition assume that the derivatives  $\dot{x}(t)$  and  $\dot{y}(t)$  exist with at most finite discontinuities over the interval  $[a, b]$ . The unit normals to the curves  $x(t)$  and  $y(t)$  are respectively defined as follows:

$$\hat{\nabla}x(t) = \frac{(-\dot{x}(t), 1)}{\sqrt{1 + \dot{x}(t)^2}}, \quad (\text{A.3})$$

$$\hat{\nabla}y(t) = \frac{(-\dot{y}(t), 1)}{\sqrt{1 + \dot{y}(t)^2}}. \quad (\text{A.4})$$

From the unit normal, two metric candidates are formed. One candidate is curve similarity, the other is called curve dissimilarity. The candidate known as curve dissimilarity closest represents the intuitive meaning of distance, since distance is a measure of difference, rather than similarity. It can be shown that when operating with unit vectors, the cosine of the angle between the two vectors is the scalar inner product and the sine of the angle between the two vectors in absolute value is the

magnitude of the vector product. Knowing this, there arises a relation between the similarity at a point in time and the dissimilarity at that same point in time :

$$(\text{dissimilarity})^2 = (1 - (\text{similarity})^2). \quad (\text{A.5})$$

Using the new definition for dissimilarity, let a new function be defined as

$$\lambda(\dot{x}, \dot{y}) = | \hat{\nabla} x(t) \times \hat{\nabla} y(t) |. \quad (\text{A.6})$$

For the remainder of this proof, the time index will be dropped, but it is understood that  $x=x(t)$  and that  $y=y(t)$ . After making the substitutions referred to in equations (4.6) and (4.7) and a few algebraic manipulations, the expression for  $\lambda'$  simplifies to the following form:

$$\lambda(\dot{x}, \dot{y}) = \frac{|\dot{x} - \dot{y}|}{\sqrt{1 + \dot{x}^2} \sqrt{1 + \dot{y}^2}} \quad (\text{A.7})$$

## Appendix B

### SHAPE METRIC APPLICATIONS

As with any new convenience, the Curve Shape Metric (CSM) is only as good as it is usable. To use the CSM with any confidence, one must have an idea of what the numbers coming out of the evaluation of the CSM mean. One way to calibrate the values that are returned by the CSM function is to start with a group of functions. This group of functions must contain functions that appear similar in shape to some functions in the group and different in shape to others. Visually one can tell which functions are similar in shape and which functions are dissimilar in shape. When the CSM is applied to the functions in all possible combinations, the CSM can be intuitively gauged. A threshold of similarity can be found heuristically, whereby when the CSM returns a value less than the threshold value, most people would say that the curves would be the same, and above this level, the curves would tend to appear dissimilar.

Assume that there exist five continuous functions on the closed interval  $[0,1]$ . Also, let these functions have continuous first derivatives for each value of  $t$  in the closed interval. Further, let these functions be defined as:

$$f_1(t) = 1/(1 + t^2), \quad (B.1)$$

$$f_2(t) = \exp(-t^2), \quad (B.2)$$

$$f_3(t) = 1 - t^2, \quad (B.3)$$

$$f_4(t) = \cos(\pi t/2) \quad (B.4)$$

$$f_5(t) = 1 - t. \quad (B.5)$$

In this example,  $f_1(t)$  has been chosen to be similar in shape to  $f_2(t)$  and  $f_3(t)$  has been chosen to be similar in shape to  $f_4(t)$ , while  $f_5(t)$  is chosen not to resemble either of the other four functions (refer to Figure B.1.) Refer to Table 1 for the shape correlation matrix concerning the system of five continuous functions. Assume that the system of five functions represent some repetitive process over a finite time period (preferably some integral multiple of the fundamental frequency of the process). Further assume that each of the five functions are the outputs of the process. If the process is

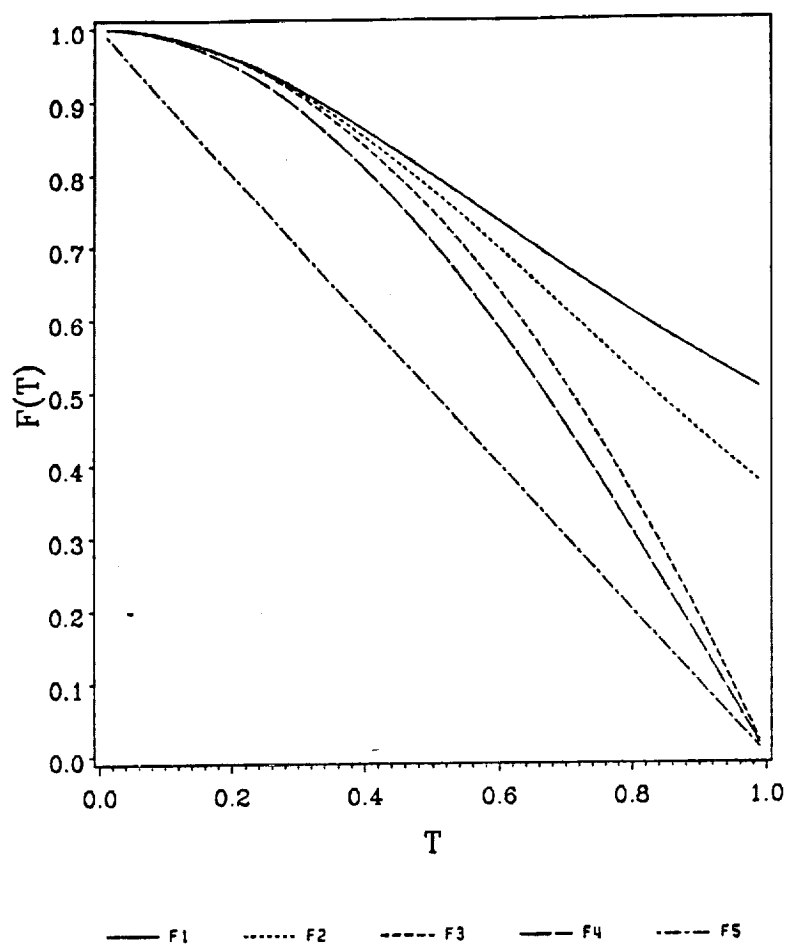


Figure B.1: Five Continuous Functions for Comparison.

Table B.1: Relative Shape Distance Among Five Functions.

	F1	F2	F3	F4	F5
F1	0.00000				
F2	0.08905	0.00000			
F3	0.24141	0.15778	0.00000		
F4	0.26336	0.17893	0.04988	0.00000	
F5	0.32004	0.23242	0.25695	0.22316	0.00000

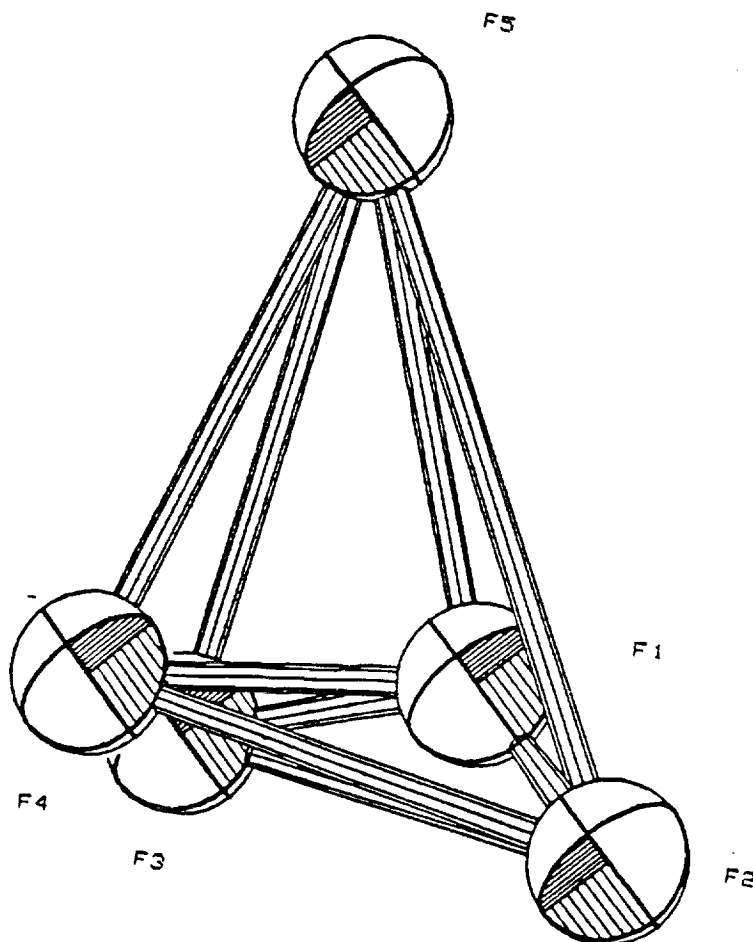


Figure B.2: Output Structure for Five Continuous Functions.

truly repetitive, whenever the shape is taken over a finite time period that has been properly chosen, the relative shapes among all the outputs  $f_1, \dots, f_5$  will tend to be constants. It is quickly noticed that the function  $f_5(t)$  is most distant from the rest of the functions in the system and tend to cause the output structure to take on the shape of a pyramid. The other functions tend to group as expected, along with the two separate pairs of functions being close together. Notice in Figure B.1, that all the functions are concave down on the interval  $[0,1]$  except  $f_5(t)$ , which causes the four to be closer together in shape space. Since the CSM is a pairwise comparison of objects, it can be mapped back into three dimensions. Thus the distances among all the outputs of the system of interest in shape space will define a construct that can be represented as a real structure in three space called the *output structure of the system* (refer to Figure B.2.) The output structure for the system of five functions adds the convenience of being able to visualize systems in shape space as a rigid structure rather than concentrating solely on a table of numbers. The CSM analysis works well on systems that are somewhat slow to change (i.e. PSL may change from hour to hour and day to day, yet when observed over a period of years, it reveals a slowly changing overall process) and is repetitive. A natural use of CSM analysis is to start with a PSL process that does not have a desirable shape (this may be because of an unwanted valley in one or more outputs that occurs periodically at a

certain time) and to observe the effect of a load management policy as it impacts upon the output structure with respect to a pre-formulated desired output structure. The change in the output structure is a measurable quantity and it can be determined if a policy caused statistically significant changes in the output with respect to a standard reference.

CSM analysis can also be used to gauge the effectiveness of a model. The possibilities of using shape analysis are boundless. Past data may be compared to future data to detect changes in the load output structure, future data may be used to check the goodness of shape of a model based upon past data.



## Appendix C

### Polar Metric Development

#### C.1 CLOSED CURVES IN $\mathbb{R}^2$

In the previous sections, the shape metric was confined to discussing functions in  $\mathbb{R}^2$ . This approach has limited use when comparing objects with varying degrees of complexity. This complexity could be in the form of angular rotations, enlargements, or translations, which could make objects similar in shape distinctly different due to its relative orientation. We enter a restriction on our shape metric that will limit its current application to object outlines. Objects which possess internal structures will not be considered. Assume there exists two objects in two-space that can be translated such that their centroids are centered about the origin (Figure C.1.) Further assume that the objects can be described by continuous mathematical relations  $C_1$  and  $C_2$  which may be represented as follows:

$$C_1 = (r_1(\theta), \theta) \quad \forall \theta \in [0, 2\pi] \quad (\text{C.1})$$

$$C_2 = (r_2(\theta), \theta) \quad \forall \theta \in [0, 2\pi] \quad (\text{C.2})$$

For all angles  $\theta$ , let the unit normals to  $C_1$  and  $C_2$  be represented by  $\hat{\nabla}C_1(\theta)$  and  $\hat{\nabla}C_2(\theta)$ . In cartesian coordinates, the unit normals are determined to be as follows:

$$\hat{\nabla}C_1 = \frac{\frac{dr_1}{d\theta} \sin \theta + r_1 \cos \theta}{\sqrt{(\frac{dr_1}{d\theta})^2 + r_1^2}} \hat{i} + \frac{-\frac{dr_1}{d\theta} \cos \theta + r_1 \sin \theta}{\sqrt{(\frac{dr_1}{d\theta})^2 + r_1^2}} \hat{j} \quad (\text{C.3})$$

$$\hat{\nabla}C_2 = \frac{\frac{dr_2}{d\theta} \sin \theta + r_2 \cos \theta}{\sqrt{(\frac{dr_2}{d\theta})^2 + r_2^2}} \hat{i} + \frac{-\frac{dr_2}{d\theta} \cos \theta + r_2 \sin \theta}{\sqrt{(\frac{dr_2}{d\theta})^2 + r_2^2}} \hat{j} \quad (\text{C.4})$$

A metric similar to the one developed for functions in  $\mathbb{R}^2$  can be developed for closed curve as well. The proposed metric, is as follows:

$$\mathcal{L}_{C_1 C_2} = \frac{1}{2\pi} \int_0^{2\pi} \|\hat{\nabla}C_1 \times \hat{\nabla}C_2\| d\theta \quad (\text{C.5})$$

The proposed metric also assumes that all target objects are to be compared to entries belonging to a standard library of objects that are already centered at the origin. All

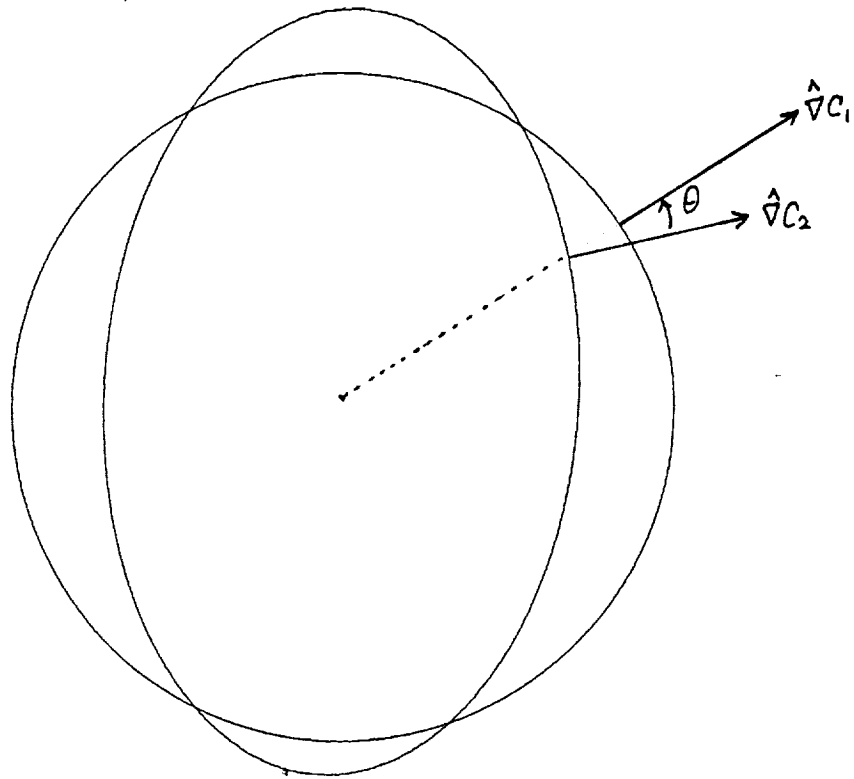


Figure C.1: Two Centered Objects in Two-Space.

comparisons will be performed by examining the normal to the curve for the following values of  $\theta$ :

$$2\pi \geq \theta \geq 0.$$

One problem that may arise is  $C_1$  and  $C_2$  are similarly shaped, but one is a rotate of the other. In a case such as this, the shape metric will give a dissimilar reading for two objects that are in fact similar rotated. To minimize errors due to rotated objects, the curve shape metric must be considered over all possible z-axis rotations as follows:

$$\mathcal{L}_{C_1 C_2} = \frac{1}{2\pi} \int_0^{2\pi} \|\hat{\nabla} C_1(\theta) \times \hat{\nabla} C_2(\theta + \alpha)\| d\theta \quad \forall \alpha \in [0, 2\pi) \quad (\text{C.6})$$

Assuming that the two objects may be rotates of each other, the shape metric must be minimized over  $\alpha$ . When the shape metric has been minimized, the angle found to be the minimum, at  $\alpha_{min}$ , represents the angle at which object two must be rotated to most closely resemble the other object.

## C.2 Conclusions

We have presented a method of identifying two-dimensional objects. The method is based upon the unit gradient of an observed plane figure and the unit of a standard reference figure over a specified range of points. Functions in two-space are decomposed into functions of a single variable. These functions are then compared with a library of functions. The curve shape metric yields a value between zero and one inclusive. These values represent similarity and dissimilarity respectively. A zero would imply a perfect match according to shape and a one would indicate that the two shapes are distinctly different. This curve shape metric also requires that objects be decomposed into functions of a single variable before they can be tested for similarity or dissimilarity. Objects under study have been limited to having no internal structures. Instead of decomposing the objects, the curve shape metric was transformed into a polar metric whereby, the decomposition of objects was handled by this transformation. With this method objects are translated so that its centroid is mapped to the origin. The object is compared with a standard reference by comparing the unit normals to the curves at every angle  $\theta$ . The resultant is a curve index

$$\mathcal{L}_{C_1 C_2} = \frac{1}{2\pi} \int_0^{2\pi} \|\hat{\nabla} C_1 \times \hat{\nabla} C_2\| d\theta \quad (\text{C.7})$$

The object is translated to consider all possible z-axis rotations which lead to the following formula:

$$\mathcal{L}_{C_1 C_2} = \frac{1}{2\pi} \int_0^{2\pi} \|\hat{\nabla} C_1(\theta) \times \hat{\nabla} C_2(\theta + \alpha)\| d\theta \quad \forall \alpha \in [0, 2\pi) \quad (\text{C.8})$$

When, the minimum of the function  $\mathcal{S}(C_1, C_2)$  is obtained, then a decision must be made to determine if the overall minimum is small enough to consider the target identifiable as a member of the standard library. Our current research involves evaluating

the feasibility of implementing this algorithm using distributed processing algorithms which possesses a parallel architecture. Methods are also being considered to handle two- dimensional objects with internal substructures.

## **Appendix D**

### **Presentation Transparencies**

The following transparencies are from presentations given by Dr. Lebby and Mr. Matherson.

RECOGNITION OF PLANE FIGURES  
USING GRADIENT BASED SHAPE METRIC

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September 23, 1988

## ITEMS FOR DISCUSSION

1. Background Information
2. Motivation of Shape Metric
3. Illustrative Examples
4. Potential Applications
5. Concluding Remarks

# Outline

## 1. Background Information

- (a) Segmentation Analysis
- (b) Measuring Surface Distortion
- (c) Shape and Distance Concepts

## 2. Motivation of Shape Metric

- (a) Extracting a Feature of Shape
- (b) Shape Metric Definition
- (c) Shape Space Concept
- (d) Extensions to Plane Figures



## Concluding Remarks

1. Exploration into three-dimensional classification using the shape metric needed.
2. Investigation into the usefulness of the shape metric as an index of surface distortion needed.
3. Development of methods to calibrate the distortion threshold level.

# Potential Applications

- Object Classification
- Surface Distortion Measurement
- Neural Network Feature Extractor

**TWO DIMENSIONAL OBJECT IDENTIFICATION USING  
A GRADIENT BASED METRIC TECHNIQUE**

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# ITEMS FOR DISCUSSION

- BACKGROUND INFORMATION
- DERIVATION OF SHAPE METRIC
- SHAPE METRIC EXAMPLES
- POTENTIAL APPLICATIONS
- CONTINUING RESEARCH
- CONCLUDING REMARKS

# BACKGROUND INFORMATION

- **PURPOSE** - To investigate techniques for identifying two-dimensional objects.
- **SCOPE** - Confined to simple convex geometrical shapes.
- **METHOD** - Shape identification using shape metric.

# POTENTIAL APPLICATIONS

- OBJECT CLASSIFICATION
- SURFACE DISTORTION
- NEURAL NETWORK FEATURE EXTRACTION

# CONTINUING RESEARCH

- EXAMINATION OF MORE COMPLEX OBJECTS
- MODEL MORE THAN SIMPLE ROTATIONS AND INVERSIONS
- METHODS TO EXAMINE INTERNAL DETAILS OF COMPLEX OBJECTS

## CONFERENCES AND PAPERS

- "POWER SYSTEM LOAD MODELING USING A METHOD OF DATA HANDLING TECHNIQUE"
- "TWO-DIMENSIONAL OBJECT DETECTION TECHNIQUE USING A GRADIENT BASED METRIC"
- 1989 IEEE SYMPOSIUM ON SYSTEM THEORY
- 1989 IEEE SOUTHEAST CONFERENCE



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